

NAG Toolbox for MATLAB

g05ga

1 Purpose

g05ga generates a random orthogonal matrix.

2 Syntax

```
[a, ifail] = g05ga(side, init, m, a, 'n', n)
```

3 Description

g05ga pre- or post-multiplies an m by n matrix A by a random orthogonal matrix U , overwriting A . The matrix A may optionally be initialized to the identity matrix before multiplying by U , hence U is returned. U is generated using the method of Stewart 1980. The algorithm can be summarized as follows.

Let $x_1, x_2, \dots, x_{n-1}$ follow independent multinormal distributions with zero mean and variance $I\sigma^2$ and dimensions $n, n-1, \dots, 2$; let $H_j = \text{diag}(I_{j-1}, H_j^*)$, where I_{j-1} is the identity matrix and H_j^* is the Householder transformation that reduces x_j to $r_{jj}e_1$, e_1 being the vector with first element one and the remaining elements zero and r_{jj} being a scalar, and let $D = \text{diag}(\text{sign}(r_{11}), \text{sign}(r_{22}), \dots, \text{sign}(r_{nn}))$. Then the product $U = DH_1H_2 \dots H_{n-1}$ is a random orthogonal matrix distributed according to the Haar measure over the set of orthogonal matrices of n . See Theorem 3.3 in Stewart 1980.

4 References

Stewart G W 1980 The efficient generation of random orthogonal matrices with an application to condition estimates *SIAM J. Numer. Anal.* **17** 403–409

5 Parameters

5.1 Compulsory Input Parameters

1: **side** – string

Indicates whether the matrix A is multiplied on the left or right by the random orthogonal matrix U .

side = 'L'

The matrix A is multiplied on the left, i.e., pre-multiplied.

side = 'R'

The matrix A is multiplied on the right, i.e., post-multiplied.

Constraint: **side** = 'L' or 'R'.

2: **init** – string

Indicates whether or not **a** should be initialized to the identity matrix.

init = 'I'

a is initialized to the identity matrix.

init = 'N'

a is not initialized and the matrix A must be supplied in **a**.

Constraint: **init** = 'I' or 'N'.

3: **m – int32 scalar**

m , the number of rows of the matrix A .

Constraints:

if **side** = 'L', $m > 1$;
 $m \geq 1$ otherwise.

4: **a(lda,n) – double array**

lda, the first dimension of the array, must be at least **m**.

If **init** = 'N' then **a** must contain the matrix A .

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **a**.

n , the number of columns of the matrix A .

Constraints:

if **side** = 'R', $n > 1$;
 $n \geq 1$ otherwise.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, wk

5.4 Output Parameters

1: **a(lda,n) – double array**

The matrix UA when **side** = 'L' or the matrix AU when **side** = 'R'.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **m** < 1,
 or **n** < 1,
 or **lda** < **m**.

ifail = 2

On entry, **side** ≠ 'L' or 'R',
 or **init** ≠ 'I' or 'N'.

ifail = 3

On entry, an orthogonal matrix of dimension 1 has been requested.

7 Accuracy

The maximum error in $U^T U$ should be a modest multiple of *machine precision*.

8 Further Comments

g05gb computes a random correlation matrix from a random orthogonal matrix.

9 Example

```
side = 'Right';  
init = 'Initialize';  
m = int32(4);  
a = zeros(4, 4);  
g05za('O');  
g05cb(int32(0));  
[aOut, ifail] = g05ga(side, init, m, a)
```

```
aOut =  
   -0.4606    0.8230   -0.2505    0.2184  
    0.4456    0.4703    0.0643   -0.7591  
   -0.7656   -0.2035    0.2563   -0.5538  
    0.0560    0.2449    0.9313    0.2636  
ifail =  
      0
```